**Proceedings of the**
**International Conference on Mechanical Engineering and Renewable Energy 2019**
**(ICMERE2019) 11 – 13 December, 2019, Chittagong, Bangladesh**

**ICMERE2019-PI-179**

# MATHEMATICAL SIMULATION OF WAVE ENERGY HARVESTING USING BUOY STRUCTURE

**Tasmia Binte Hai**, **Md. Arafat Rahman***, **Shihab Shahriar Kabir**, **Salmiah Akhtar**

Department of Mechanical Engineering,
Chittagong University of Engineering and Technology, Chattogram- 4349,
Bangladesh

E-mail: tasmiaheem.th@gmail.com ; arafat@cuet.ac.bd*; showmikmb@gmail.com ;
salmiahsara12@gmail.com

***Abstract***- *Wave energy is reliable source for power production from the perspective of certainty which can be harvested in many ways. Buoyancy force serves as the most effective and simple method for harvesting wave energy. However, the main concern of this work is to find out effective buoy structure of specific material for harvesting wave energy in a catchment area. This study included designing of different buoy as well as simulating these individually and compared among these structures. In addition, a mathematical modeling for angles as well as forces acting on these buoy geometries is presented here.*

**Keywords:** Wave energy, Buoy structures, Design, Simulation, Mathematical modeling

## 1.INTRODUCTION

Highly developed countries are showing an increasing demand for energy production. In addition, the fossil fuel depletion, ecological requirements and safety policies of environment are claiming for the dependency on renewable energy sources [1].Wave energy can serve better as it is always formed in ponds, rivers, seas regardless of time and weather. As the wind blows across the water surface, it transfers the energy of winds to waves. The wave speed has to be slower than wind above the water surface for this transmission [2]. The friction and the pressure difference between the upwind and the water surface make the water going into sheer stress. And this sheer stress causes the growth of the waves [3]. The energy output is measured by wave speed, wave height, and wave length and water density [4].

Wave energy technologies have just started to reach as potential commercial power sources. The technology is advancing rapidly for generating power. At present there are existing a few projects regarding wave energy. Recently many countries have proposed to use wave energy or some are using wave power plants. According to the recent situation of our country Bangladesh, we still have not proposed any wave power plant yet. Bangladesh has a population density of 1,115.62 people per square kilometer (2,889.45/square mile), which ranks 10th in the world [5]. Solar power plant cannot be an effective solution of power generation for us as we have an acute shortage of space. Wind power plant results in uncertainty. As a result, comparing with solar and wind power plant conditions, we can see that in our country, we have a very positive possibility of producing power from wave energy.

Wave energy can be harvested very successfully by using buoyancy forces. A body experiences an upward force, opposite to gravitational force while it is immersed partially or wholly in a fluid. This force provides a lift to the body. This upthrust force is called buoyancy force [6]. Buoy structures (a floating structure or device) are needed to utilize this buoyancy force. The buoy structure transfers the wave energy from the water element to the mechanism. Generally, buoys can be of various geometries. They can be (a) cylindrical, (b) spherical (c) conic and (d) tulip. The tulip geometrical structure is the combination of cylindrical shape at the upper section and conical shape at the lower section. These buoys face a hydrodynamic force, when they are submerged [7].

In general polyurethane material is used as the core of the buoy which are used in commercial purpose. Including, the shell is made of polyethylene of high density [8]. In this paper we have used poly vinyl chloride rigid (PVC) as design material for all the structures.

### 1.1 Objective of the Study

Objectives of the study are given below:

- To design various buoy geometries of specific volume and material.
- To run simulation of these buoy structures.
- To illustrate a thorough comparison
- To observe the most effective structure using finite element method.

## 2. METHODOLOGY

Poly vinyl chloride rigid material has been selected for designing and simulation purpose of this paper. I have chosen the material as it is very much available and cost effective. In addition, this material consists of resistant to corrosion, chemical rotting, abrasion, and weathering. It is very light in weight. The mechanical properties of PVC rigid is given in the table 2.1.

Table 2.1: Mechanical properties of PVC rigid

| Properties | Value |
|---|---|
| Density | 1380 kg/m³ |
| Young's modulus | 2900-3300 MPa |
| Tensile strength | 50-80 MPa |
| Elongation at break | 20-40% |
| Impact strength | 2-5 kJ/m² |
| Melting point | 212˚C |
| Heat transfer coefficient | 0.16 W/m.K |
| Water absorption | 0.04-0.4 |
| Specific heat | 0.9 kJ/(kg.K) |
| Linear expansion coefficient | $8.10^{-5}$ /K |

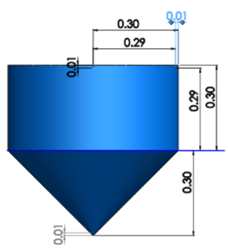
Different buoy geometries have been designed in solidworks. The volume have been kept same for all the structures. The comparison has been easier due to same volume of these structures. Further these buoy structures have been simulated under finite element method. Boundary conditions have been applied. This structures have been run through equivalent stress (von misses) analysis, directional deformation analysis. A mathematical modeling has been derived to represent a relationship between buoyancy force and displacement.

### 2.1 Assumption

- Only gravitational force and buoyancy force has been considered for mathematical modeling.
- Slamming force has been neglected.
- No relative motion at the joint of the buoy and mechanism.

### 2.2 Buoy Structure Design

The buoy geometries which have been showed in figure 1.2 have been designed.



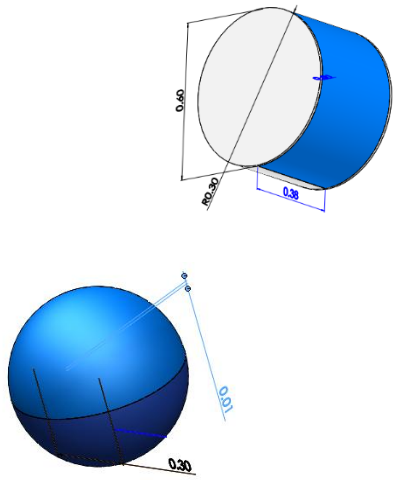




Figure 2.1: CAD model of buoy geometries

The volume of each structure has been kept 0.1131m³. Each buoy has hollowed structure inside. The thickness of every wall has been kept 0.01m. Hence there have been an accurate comparison.

### 2.3 Mathematical Simulation

The same boundary condition have been applied to each buoy structure. We have run equivalent stress analysis and deformation analysis over these structures. The deformation analysis has been placed to observe the maximum displacement of each buoy structure. The displacements has been kept same for each structure. The displacement data have been input in tabular form as well as only towards the vertical direction opposite to the gravitational force. Further the deformation simulations have been performed. Each buoy structure has shown different deformation as result. The maximum deformation shown by the buoy structure would be considered to experience maximum displacement. The only forces acting on the buoy structure has been considered to be gravitational force and buoyancy fore. Both of these forces acts opposite to each other. It has been taken in account while applying load condition for stress analysis and deformation analysis. The forces which have been applied on positive vertical direction are 10000N-60000N.

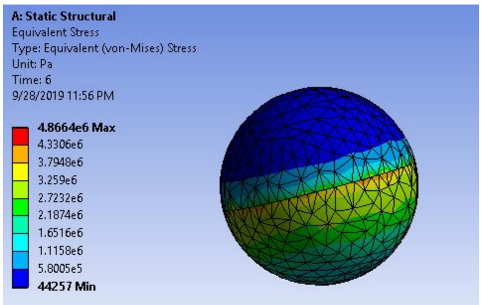



Figure 2.2: Static structural analysis of equivalent stress (Von- Misses) PVC sphere buoy

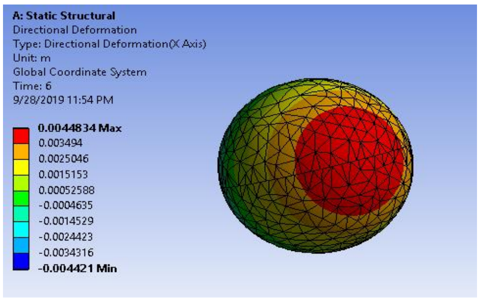



Figure 2.3: Static structural analysis of deformation of PVC sphere buoy

We have observed the maximum stress to be $4.86 \times 10^6$ N/m² for spherical PVC buoy structure using ansys. It is less than the value of ultimate strength of the material. In addition, the directional deformation has been observed to be 0.0044834m.

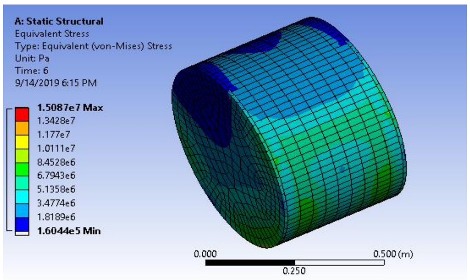



Figure 2.4: Static structural analysis of equivalent stress (Von-Misses) of PVC cylindrical buoy.

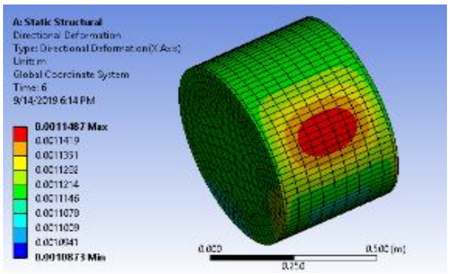



Figure 2.5: Static structural analysis of deformation of PVC cylindrical buoy

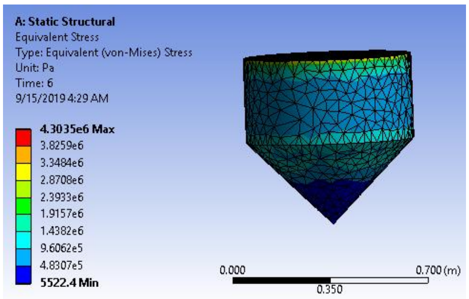



Figure 2.6: Static structural analysis of equivalent stress (Von- Misses) PVC tulip buoy

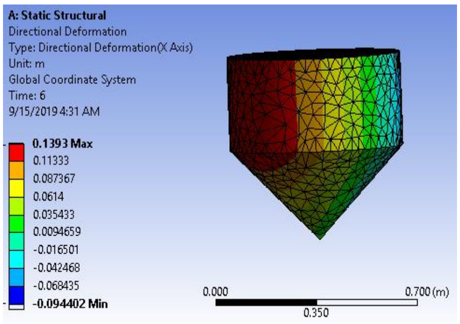



Figure 2.7: Static structural analysis of deformation of PVC tulip buoy

We have observed the maximum stress to be $4.3035 \times 10^6$ N/m² for tulip shaped PVC buoy structure. This value is smallest among all the maximum stresses. The maximum directional deformation is 0.1393m. This is the maximum deformation which has been observed in tulip shaped buoy structure.

We have observed the maximum stress to be $4.3035 \times 10^6$ N/m² for tulip shaped PVC buoy structure. This value is smallest among all the maximum stresses. The maximum directional deformation is 0.1393m. This is the maximum deformation which has been observed in tulip shaped buoy structure.

## 3. Mathematical Modeling

The buoys are always analyzed following the Archimedes principle. The buoy faces a buoyancy force towards the upward direction when it is immersed partially or fully in water. The body displaces some amount of fluid. The weight of this displaced fluid is equal to the buoyancy force. The buoyancy force ($F_b$) has to be equal or greater than the gravity force ($F_g$) to make the buoy float.

Hence to make the buoys float, $F_b \geq F_g$

The gravitational force generally also includes the slamming $F_s$. But here the slamming force is not considered. Hence, $F_s = 0$ and $F_g = m\ddot{x}$, ($\ddot{x}$ = acceleration force due to gravity, m = mass of the buoy body).

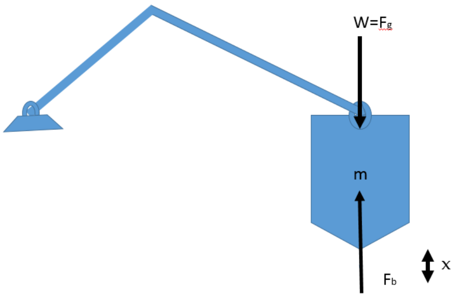



Figure 3.1: experimental setup

The up warded buoyancy force, $F_b = \rho_b \times g \times V_b$

For equilibrium condition, the weight of the buoy is equal to the weight of the water displaced. Hence,

In equilibrium, volume of the spherical buoy, $V_b = \frac{4}{3}\pi r_b^3$

In equilibrium, volume of the cylindrical buoy, $V_b = \pi r^2 h$

In equilibrium, volume of the conical buoy, $V_b = \pi r^2 h/3$

In equilibrium, volume of the tulip buoy, $V_b = (\pi r^2 h_1 + \pi r^2 h_2/3)$

Considering the tulip buoy is not in equilibrium,

Volume of the tulip buoy in water, $V_b = (\pi r^2 h_3 + \pi r^2 h_2/3)$

Here, $h_3$= height of the cylindrical buoy in water and,

$F_b = -F_g$

$F_b + F_g = 0$

$\rho_b \times g \times V_b + m\ddot{x} = 0$

$\rho_b \times g \times (\pi r^2 h_3 + \pi r^2 h_2/3) + m\ddot{x} = 0$

$\rho_b \times g \times \pi r^2 (h_3 + h_2/3) + m\ddot{x} = 0$

Let, $(\rho_b \times g) = k$ and $(h_3 + h_2/3) = h_x$

$k \times \pi r^2 \times h_x + m\ddot{x} = 0$

$k/m \times \pi r^2 \times h_x + \ddot{x} = 0$

$K \times \pi r^2 \times h_x + \ddot{x} = 0$

$\ddot{x} = -K \times \pi r^2 \times h_x$

By double differentiating,

$x = -\partial^2(K\pi r^2 \times h_x)$

Now taking the angle of float,

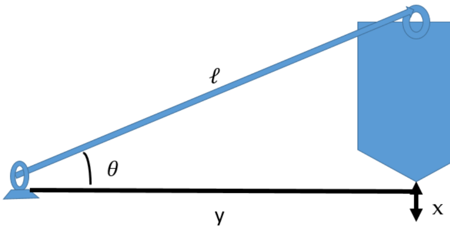



Figure 3.2: experimental setup

From the law of Pythagoras, $\ell^2 = y^2 + x^2$

Or, $x^2 = \ell^2 - y^2$

Or, $x = \sqrt{(\ell^2 - y^2)}$

Again, $\sin\theta = {}^{x}/_{\ell}$

Or, $x = \ell \sin\theta$

And, $\theta = \sin^{-1}(x/\ell)$

## 4. RESULTS AND DISCUSSION

The table 4.1 represents the maximum equivalent stress (Von-Misses) and maximum directional deformation of each buoy for an easier comparison. In addition, it has included the normalized graphical analysis of the simulation in figure 4.1, 4.2 and 4.3

Table 4.1: Properties of buoy structures

| Buoy structures | Maximum equivalent stress ( $N/m^2$) | Maximum directional deformation |
|---|---|---|
| Cylindrical | $1.51 \times 10^7$ | 0.0011487m |
| Spherical | $4.86 \times 10^6$ | 0.0044834m |
| Tulip | $4.30 \times 10^6$ (min) | 0.1393000m (max) |

The discussion above has represented that tulip shaped buoy has the minimum equivalent stress which is $4.30 \times 10^6 N/m^2$. Moreover, the tulip buoy experiences the maximum deformation. This maximum deformation indicates to the maximum possible displacement among all these buoy structures.

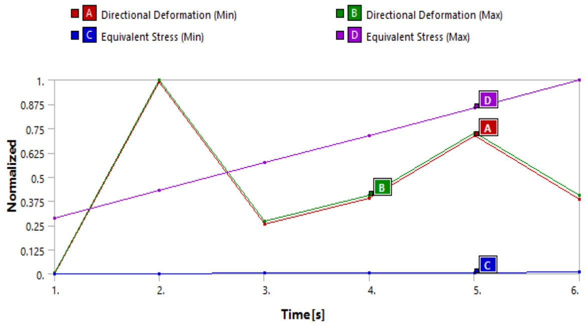



Figure 4.1: Graphical analysis of cylindrical buoy

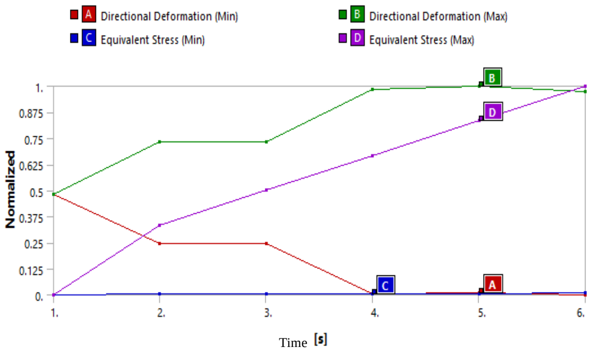



Figure 4.2: Graphical analysis of spherical buoy

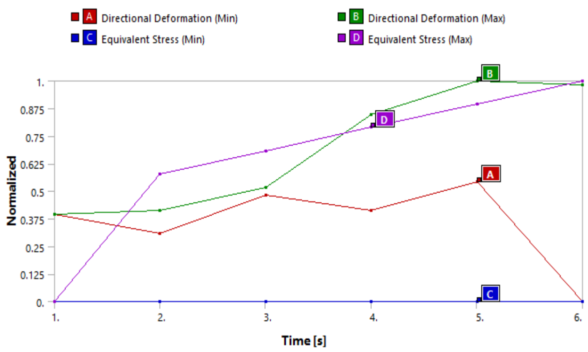



Figure 4.3: Graphical analysis of tulip buoy

The horizontal axis represents the time and the vertical axis normalizes the equivalent stress (MPa) and deformation (m). According to the applied boundary condition load increases linearly with the increasing time. The tulip buoy experiences maximum equivalent stress (minimum) nearly at $5^{th}$ second. The cylindrical buoy also represents same condition within the same duration. However, the tulip buoy faces maximum deformation than cylindrical buoy. Hence it has been observed that tulip buoy structure is the most effective buoy structure than cylindrical and spherical buoy structure. This statement has proved its validity for PVC plastic material. Hence this structure should be used to harvest wave energy with the help of buoy effect.

## 5. CONCLUSION

Tulip buoy structure serves best among all the buoy structures. In this paper the comparison were made considering the poly vinyl chloride rigid as buoy material.

Tulip buoy structure consists of less stress and maximum displacement due to its geometry than spherical or cylindrical buoy. The mathematical modeling has been done by considering the Archimedes principle. The theoretical calculations have been done in view of gravitational force and buoyancy force. The slamming force has not been taken into account. The derivations have been developed after assuming the system not to be in equilibrium condition. The equations have been calculated for tulip buoy structure only. However, the equations can be calculated by considering other geometries of buoy structure (spherical, cylindrical and conic) also. The theoretical derivation has developed equations for calculating the vertical displacement of the float structure and the angle between link and horizon axis on the water surface.

## 6. REFERENCES


[1] Leijon M. et. Al.:An electrical approach to wave energy conversion. Renewable Energy 31(2006) 1309-1319.

[2] Collins and Abayomi, Ocean wave energy –An option for Nigerian power situation, Volume 8(25), scientific research and essays, September, 2013

[3] Collins and Abayomi, Ocean wave energy –An option for Nigerian power situation, Volume 8(25), scientific research and essays, September, 2013

[4] Leszek Chybowski and Boleslaw Kuznniewski, An overview of methods for wave energy conversion, scientific journals of Maritime University 2015, 41(113), 17-23

[5] world ppulation review,bangladesh(12Feb,2019), http://worldpopulationreview.com/countries/bangladesh-population

[6] R.K. Rajput, “ A textbook of fluid mechanics and hydraulic machines”, edition: 2006, p-158

[7] C. Blommaert Composite floating ‘point absorbers’ for wave energy converters: Survivability Design, Production Method and Large Scake Testing Ph.D. Thesis, Ghent University, Belgium, June, 2009

[8] Pedro Beirao and Candida Malca, “Design and analysis of buoy geometries for a wave energy converter”, in 5th International Congress on Energy and Environment Engineering and Management, Lisbon, Portugal, July 17, 2013.


## 7. NOMENCLATURE

| Symbol | Meaning | Unit |
|---|---|---|
| $\rho_b$ | Density of the buoy material | $(Kg/m^3)$ |
| $V_b$ | volume of water displaced by the body | $(m^{3})$ |
| $r_b$ | Radius of the sphere | (m) |
| $h$ | Height of the cylindrical or conical buoy | (m) |
| $h_1$ | Height of the cylindrical section of tulip buoy | (m) |
| $h_2$ | Height of the conical section of tulip buoy | (m) |
| $g$ | Gravitational acceleration | $(m/s^2)$ |
| $\ell$ | length of the link | (m) |
| x | displacement of the float | (m) |
| $\theta$ | angle between link and vertical axis | (degree) |
| y | distance between float and link base | (m) |